The averaging bias – a short explanation

or ask your tax consultant …

The presentation is online: www.ifkm.kit.edu

Thomas Koch

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EDITOR'S CHOICE

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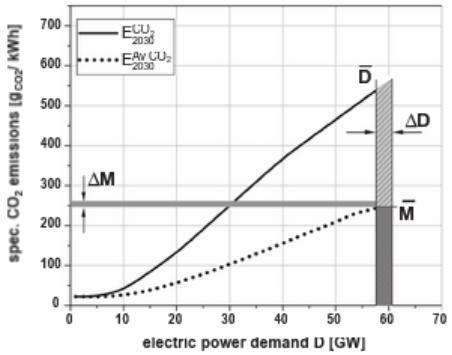
The averaging bias - A standard miscalculation, which extensively underestimates real CO_2 emissions

Thomas Koch[1] | Thomas Böhlke[2]

FIGURE 6 Graphical illustration of Equations (50) and (51) Please note that the depicted areas represent $\bar{M}(\bar{D})\Delta D$ and $\bar{D}\Delta M(\bar{D}, \Delta D)$



Motivation (example)

Und wie viel CO2 sparte das ein? Das kommt dann vor allem auf den Strommix an, mit dem man diese E-Autos lädt. Geht man davon aus, dass Elektroautos übers Jahr gesehen ungefähr den Durchschnittsstrom laden, ist ihr indirekter Klimafußabdruck leicht zu errechnen: Man benötigt ihren Verbrauch (in kWh pro Kilometer), und die Menge an CO2, die im Jahresmittel mit dem deutschen Kraftwerkspark pro kWh anfällt.

Im Kraftwerkspark gibt es Wind- und Fotovoltaikanlagen sowie Wasserkraft, Atomkraft und Biogas, also CO2-arme Erzeuger, ebenso wie Öl-, Gas- und Kohlekraftwerke. 2020 entstanden im Mittel laut vorläufigen Daten des Umweltbundesamts 366 Gramm pro Kilowattstunde deutschen Stroms, immerhin 400 Gramm oder 52 Prozent weniger als noch 1990. Bei einem durchschnittlichen Verbrauch der E-Autos auf 100 Kilometer von 18 kWh macht das rund 66 Gramm CO2 je Kilometer oder 6,6 Kilogramm auf 100 Kilometer.

Source: typical calculation with simplified approach

Anstieg CO_{2e} [g/h]= M · ΔD

66[g/km]=366 [g/kWh] · 18 [kWh/100km]

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Simplified formula (underestimates CO_2-emissions)

An example for such a simplified formula to analyze the additional CO_2 emissions per time interval $\Delta F(\bar{D}, \Delta D)$ caused by additional electric power ΔD (unit: Watt) is the direct utilization of the average CO_2 emission footprint $M(\bar{D})$ (unit g_{CO_2}/kWh) for a given average electricity demand $\bar{D}$ of the electricity sector by the equation

$$\Delta F(D, \Delta D) \approx M(\bar{D})\Delta D, \tag{49}$$

ΔF increase of CO_{2e} [g/h]= M · ΔD

Exact formula

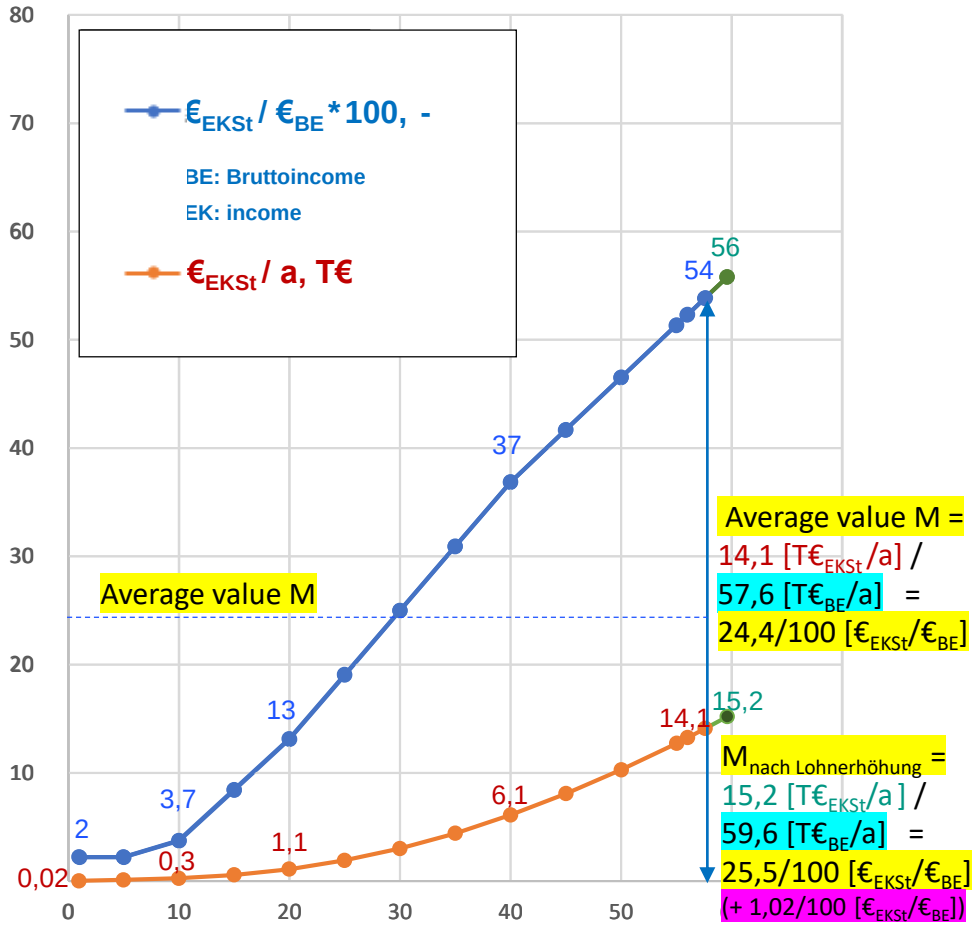
By applying the fundamental theorem of differential and integral calculation of Leibniz of the 17th century, the general and exact formula can be written as follows (see Equations (36) and (38))

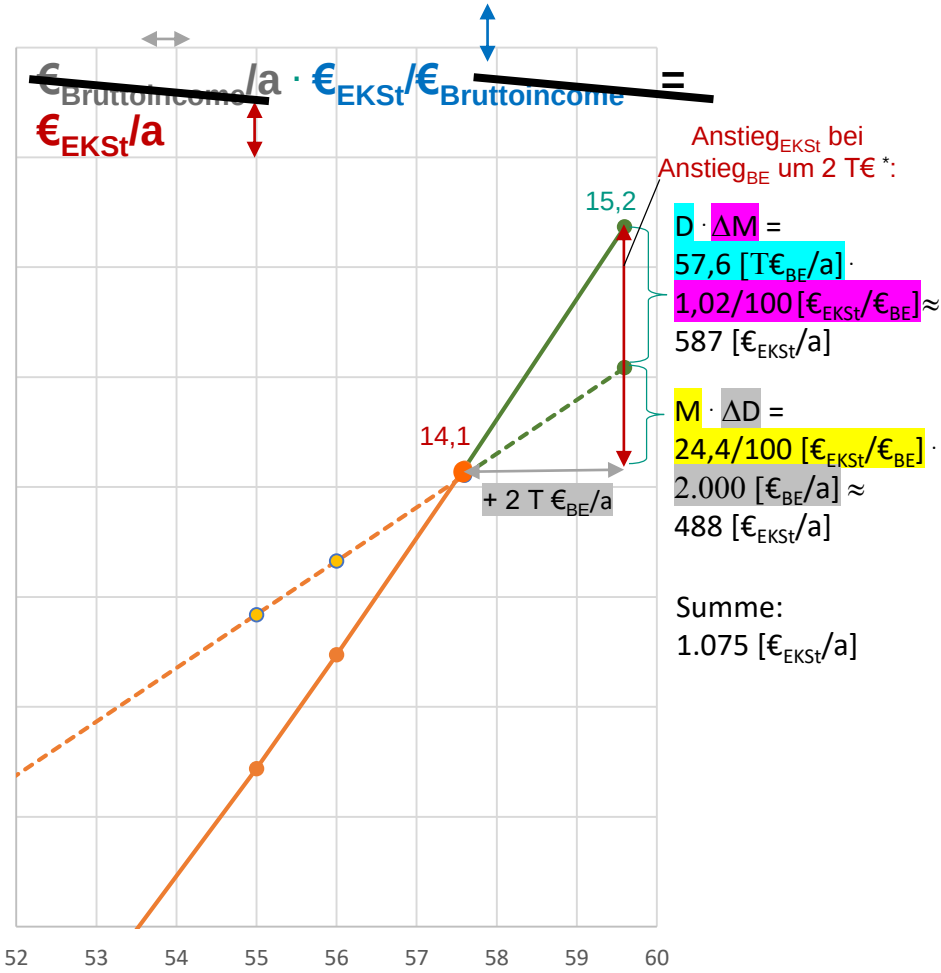
$$\Delta F(\bar{D}, \Delta D) = \bar{D}\Delta M(\bar{D}, \Delta D) + \Delta D M(\bar{D} + \Delta D). \tag{51}$$

ΔF increase of CO_{2e} [g/h]= D · ΔM + M · ΔD

The approach M · ΔD significantly underestimates the real increase of CO_{2e} emissions.

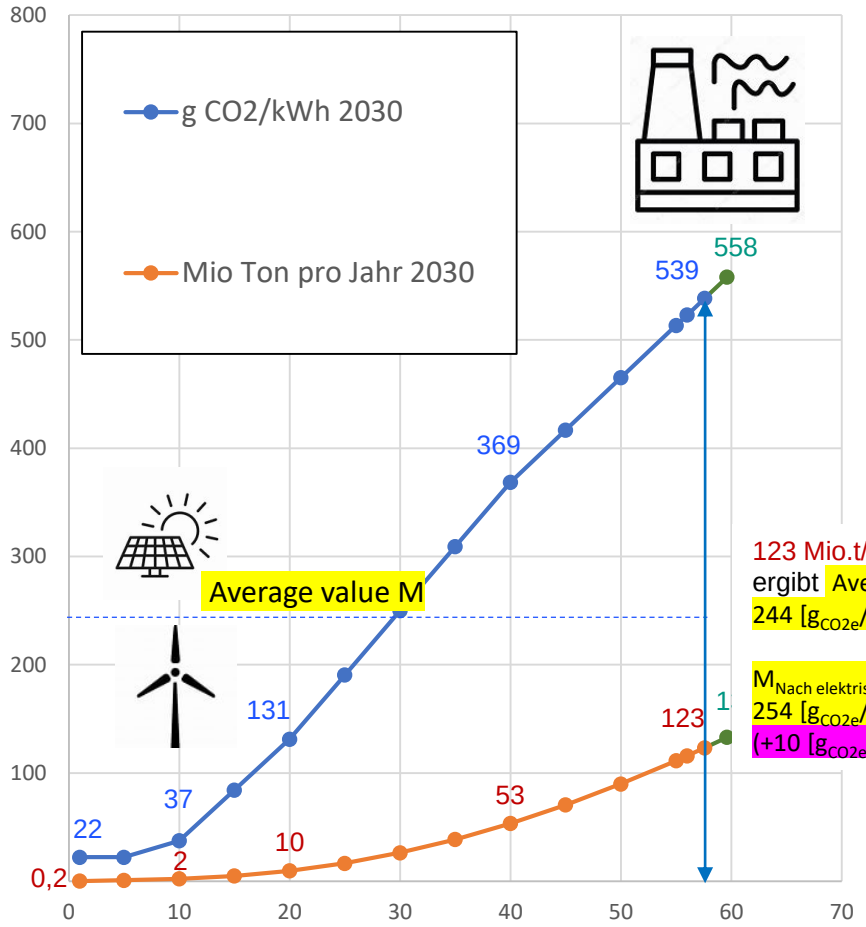
Explanation of averaging bias





- Even the shown approach M · ΔD + D · ΔM slightly underestimates the increased tax of 1097 €. This is explained in detail int the extended presentation (https://www.ifkm.kit.edu/152.php#block1961 , S.15 negligible error) and is caused by the increase of tax-rate and income, which has to considered for M und D. This mathematical details not important for the general understationg of the „averaging bias“ and can be neglected.

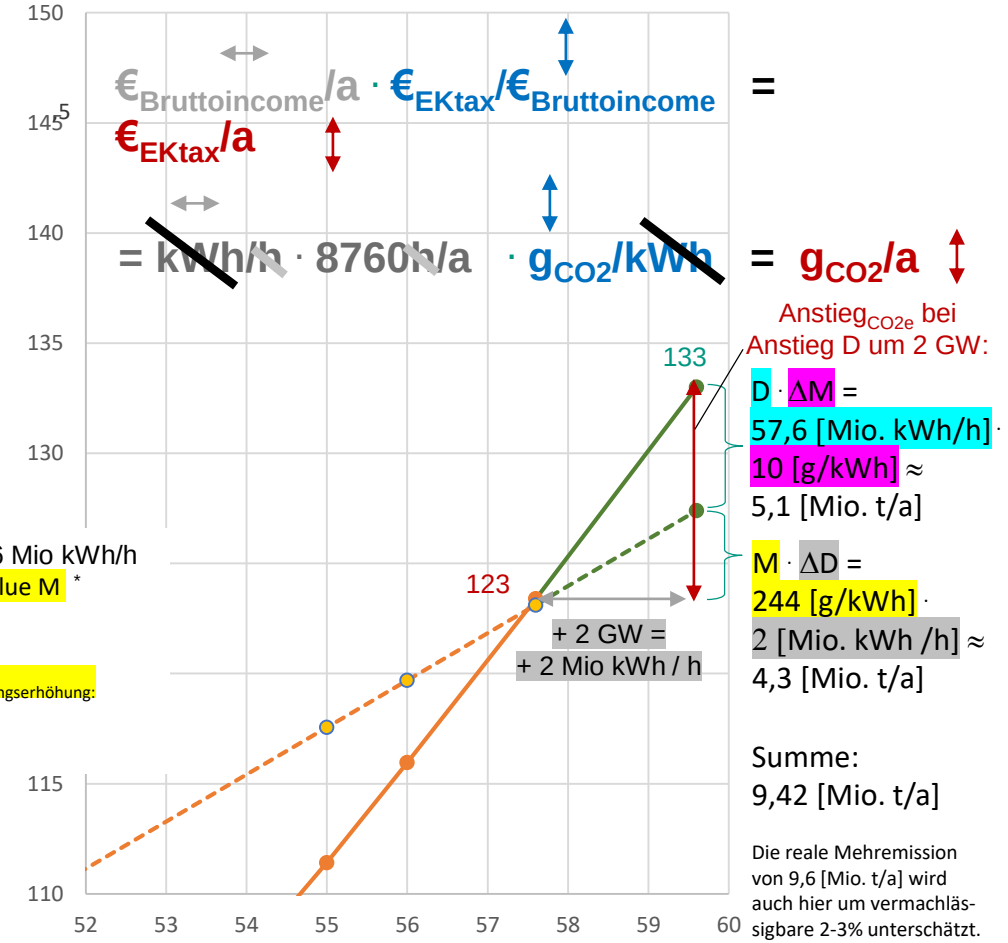
Explanation of averaging bias



123 Mio.t/a / 57,6 Mio kWh/h ergibt Average value M * 244 [g_{CO2e}/kWh]

$M_{Nach\ elektrischer\ Leistungserhöhung:}$ 254 [g_{CO2e}/kWh] (+10 [g_{CO2e}/kWh]

Average electrical load D GW; GWh/h; Mio. kWh/h



Anstieg$_{CO2e}$ bei Anstieg D um 2 GW:

D · ΔM = 57,6 [Mio. kWh/h] · 10 [g/kWh] ≈ 5,1 [Mio. t/a]

M · ΔD = 244 [g/kWh] · 2 [Mio. kWh /h] ≈ 4,3 [Mio. t/a]

Summe: 9,42 [Mio. t/a]

Die reale Mehremission von 9,6 [Mio. t/a] wird auch hier um vermachlässigbare 2-3% unterschätzt.

Average electrical load D GW; GWh/h; Mio. kWh/h

- Please watch the units, when transfering 123 Mio t CO2/a and the average value of 244 g_{CO2}/KWh as
- 8.760 h/a need to be considered.

Summary

Simplified formula (underestimates CO_2-emissions)

An example for such a simplified formula to analyze the additional CO_2 emissions per time interval $\Delta F(\bar{D}, \Delta D)$ caused by additional electric power ΔD (unit: Watt) is the direct utilization of the average CO_2 emission footprint $M(\bar{D})$ (unit g_{CO_2}/kWh) for a given average electricity demand $\bar{D}$ of the electricity sector by the equation

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ΔF increase of CO_{2e} [g/h] = D · ΔM + M · ΔD

The approach M · ΔD significantly underestimates the real increase of CO_{2e} emissions and delivers wrong results as a consequence. The real increase of CO_{2e} emissions in Germany is roughly 2,1 times higher than the result of (M · ΔD)!
You can ask your tax consultant …

Many thanks for your attention!

Additional information can be found:

https://www.ifkm.kit.edu/152.php#block1961